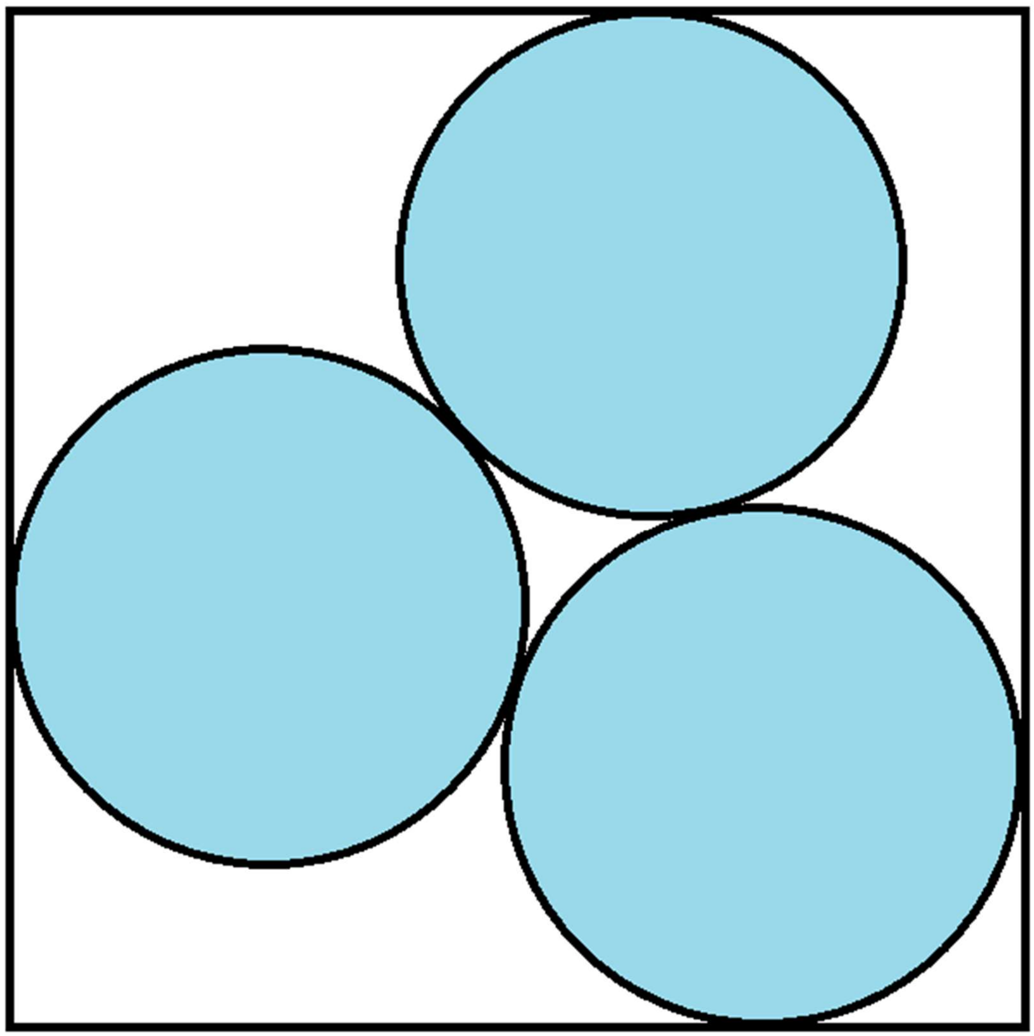
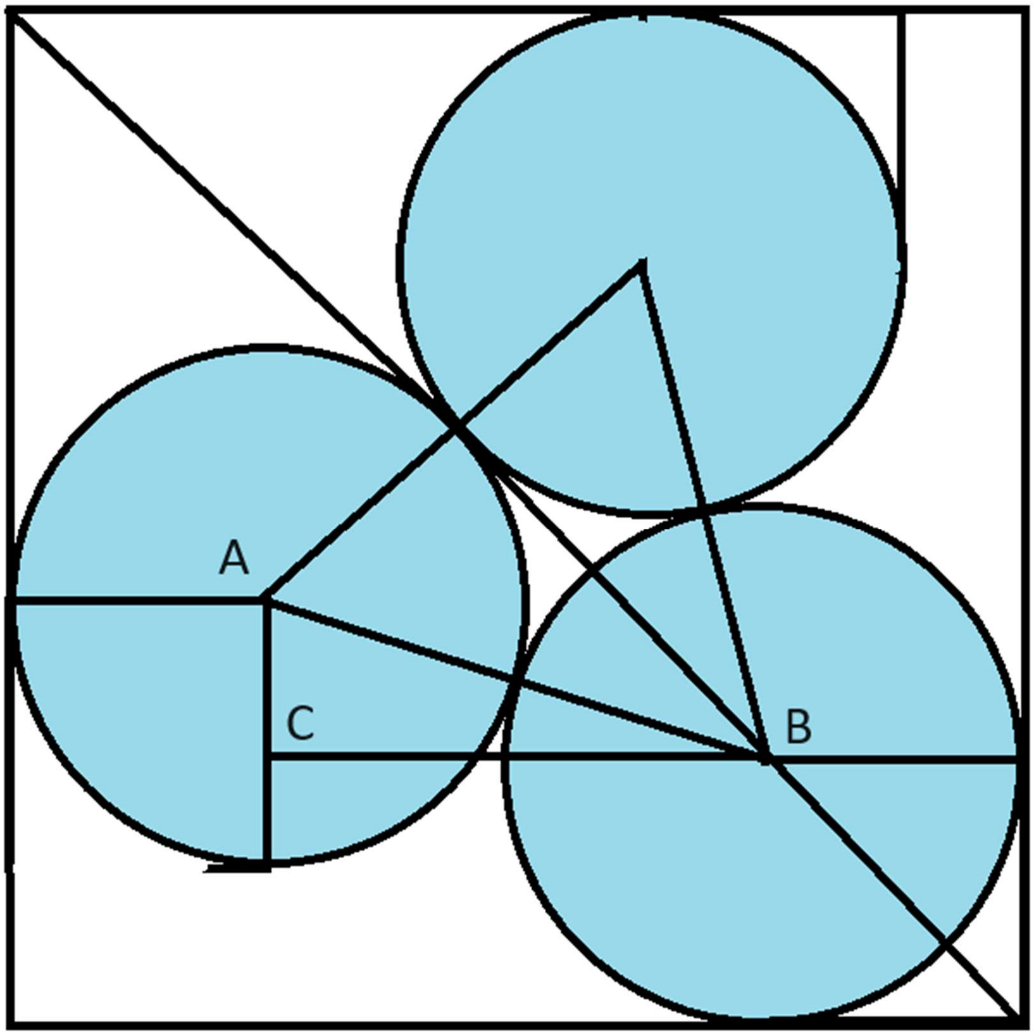
Question: In the diagram below, the three circles have radius 1. They are inscribed in a square of the least possible size. What is the length of the side of the square?



Answer: $2+\frac{\sqrt{2}+\sqrt{6}}{2}$ =~ 3.931852

Solution:

Consider the diagram below. The diagonal line of the square is supposed to be straight, but my artwork is imperfect.




Consider the triangle ABC.

AB = 2, because it consists to the radius of two of the circles.

Let t = Angle ABC = 45° – 30° = 15°

Cos(15) = BC/AB = BC/2

BC = 2×cos(15) =~ 1.931852

The rest of the side consists of the radii of two of the circles.

Thus the full side length is 2 + 2×cos(15) =~ 3.931852

However, let's express without the cos(15) part.

Recall:

Cos(x-y) = cos(x) × cos(y) + sin(x) × sin(y)

Cos(15) = Cos(45-30) = cos(45) × cos(30) – sin(45) × sin(30)

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} * \frac{1}{2}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

The answer is thus: $2 + \frac{\sqrt{6}+\sqrt{2}}{2}$