

Question: What is the average number of spins in double-zero roulette to see each outcome at least twice?

Answer: The answer is approximately 234.8326629288898.

Solution: The key to my solution is to think of spins occurring at a moment in time, with the time between spins having an exponential distribution with a mean of one unit of time. Then rephrase the question as “What is the average units of time until every number has appeared at least twice?” The answer will come out the same.

After  $t$  units of time, the probability that any given number has not appeared yet is  $e^{-t}$ . Using the Poisson distribution, the probability that any given number has appeared exactly once is  $te^{-t}$ . So, the probability a number has appeared twice or more is  $1 - e^{-t}(1+t)$ .

The probability that 38 different number have appeared at least twice is  $(1 - e^{-t}(1+t))^{38}$ .

The probability of the alternative, that at least once number has not appeared twice yet is  $1 - (1 - e^{-t}(1+t))^{38}$ .

To get the expected time of any random event, we integrate the probability from 0 to infinity of the probability the event has not happened yet. In other words, the answer can be expressed as:

$$\int_0^{\infty} 1 - (1 - e^{-t}(1+t))^{38} dt$$

I recommend the integral calculator at [www.integral-calculator.com](http://www.integral-calculator.com) to solve it. It gives the answer of apx. 234.8326629288898.

Here are the answers for single-zero, double-zero and triple-zero roulette for requiring every number be observed a minimum of one, two and three times:

Single-Zero Roulette:

At least once: 155.458690

At least twice: 227.513340

At least thrice: 290.543597

Double-Zero Roulette:

At least once: 160.660277

At least twice: 234.832663

At least thrice: 298.396127

Triple-Zero Roulette:

At least once: 165.888179

At least twice: 242.181868

At least thrice: 308.880287