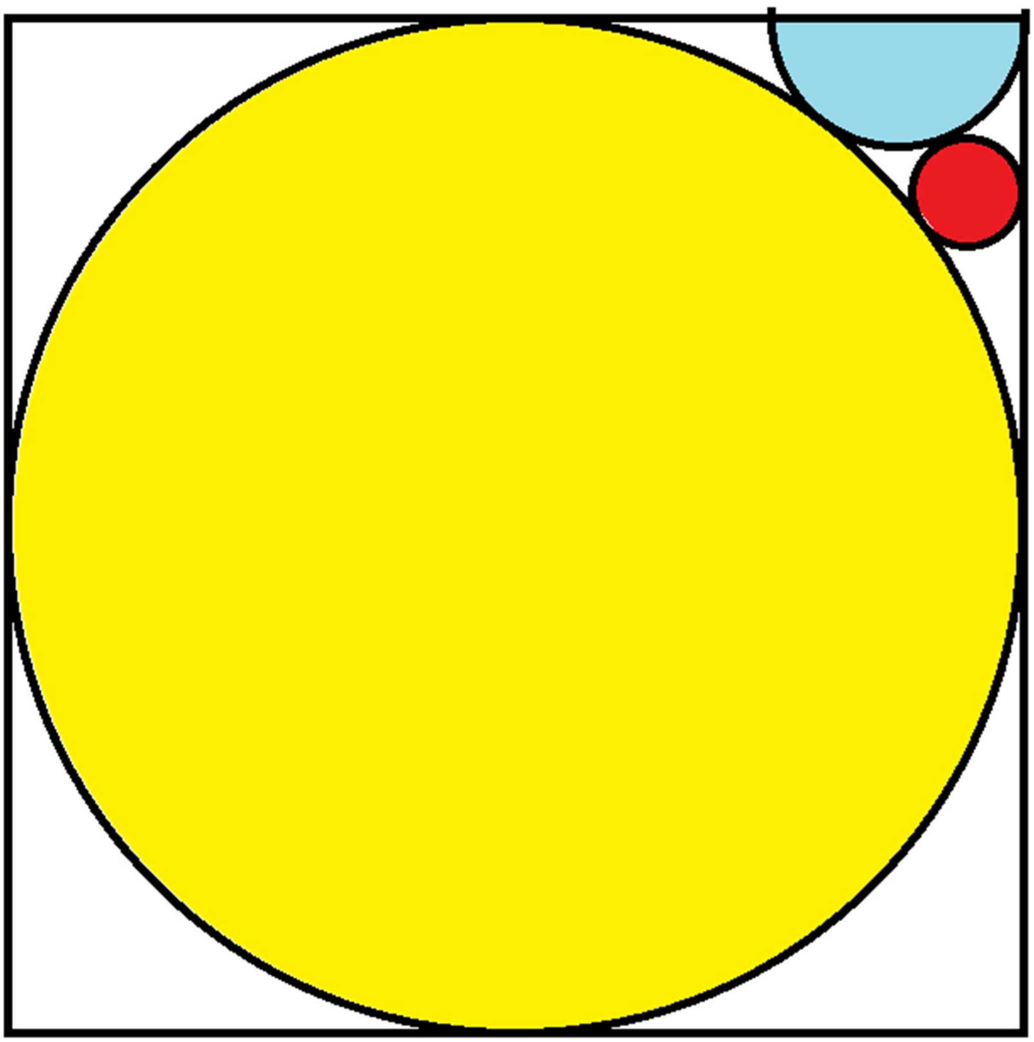
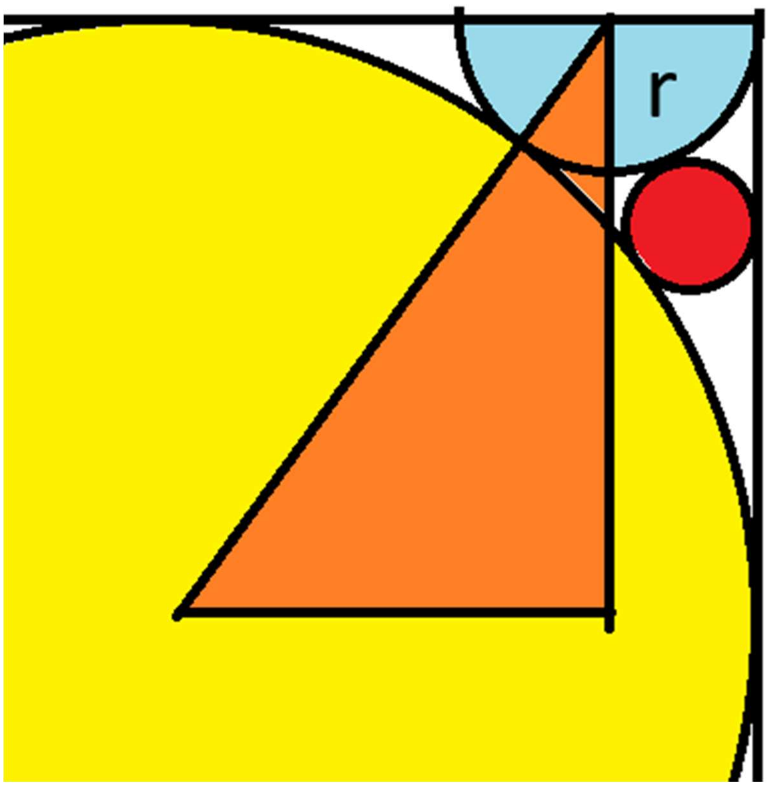
Question: The yellow circle has a radius of 1. What is the area of the red circle?



Answer: $\pi/81$ =~ 0.038785

Solution:

Let's first find the radius of the blue semicircle. To do that, consider the orange triangle below.



The sides of the orange triangle are:

Length = 1-r

Height = 1

Hypotenuse = 1+r

The Pythagorean formula tells us:

$1 + (1-r)^2 = (1+r)^2$

$1 + r^2 - 2r + 1 = r^2 + 2r + 1$

$4r = 1$

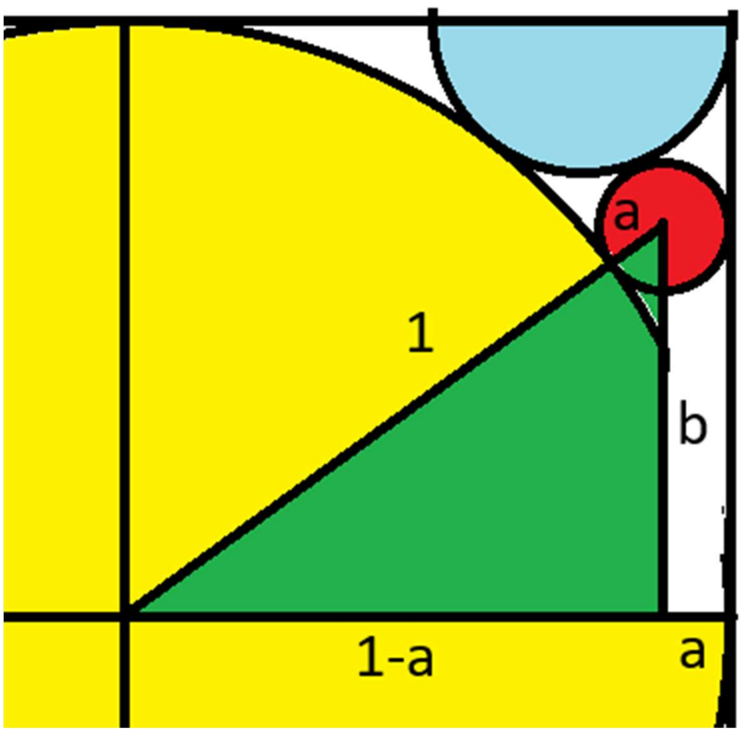
$r = 1/4$

Second, consider the green triangle as shown in the following close-up of the upper right quadrant of the yellow circle.

Let's call:

a = radius of red circle

b = height of green triangle



The Pythagorean formula tells us:

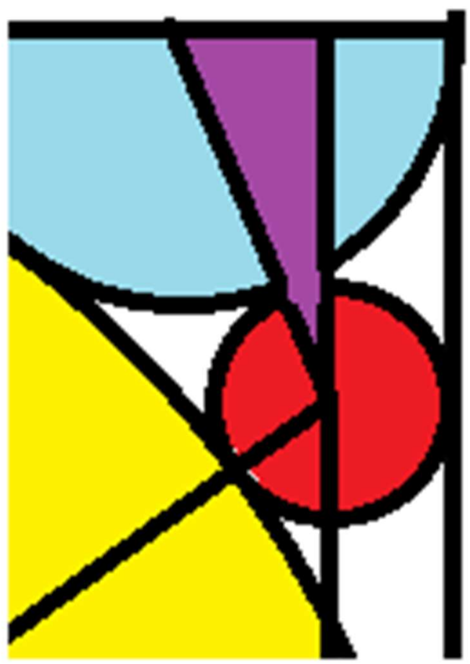
$(1-a)^2 + b^2 = (1+a)^2$

$a^2 - 2a + 1 + b^2 = a^2 + 2a + 1$

$b^2 = 4a$

$b = 2\sqrt{a}$

Next consider the purple triangle in the figure below.



The sides of the purple triangle are:

Length = $\frac{1}{4} - a$

Height = $1 - 2\sqrt{a}$

Hypotenuse = $\frac{1}{4} + a$

The Pythagorean formula tells us:

$(1/4 - a)^2 + (1 - 2\sqrt{a})^2 = \left(\frac{1}{4} + a\right)^2$

Expanding the expressions and cancelling out terms leads to:

$3a - 4\sqrt{a} + 1 = 0$

$3a + 1 = 4\sqrt{a}$

Squaring both sides:

$9a^2 + 6a + 1 = 16a$

$9a^2 - 10a + 1 = 0$

The quadradic formula tells us a = 1 or 1/9. 1/9 is the only reasonable answer.

The area is $\pi (1/9)^2$