Q: Assuming a fair coin is flipped starting with heads face up. What is the probability the coin will land on heads?

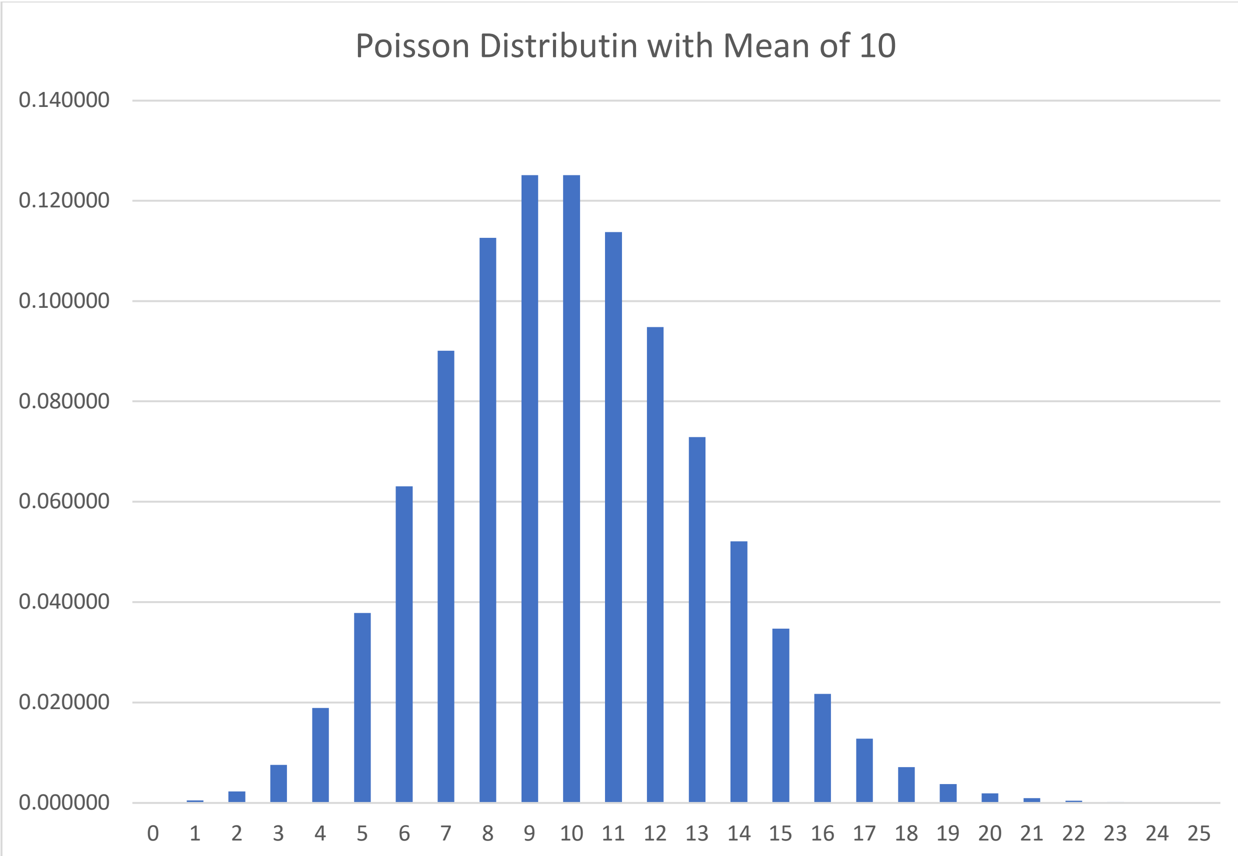
A: This depends on the way the coin is flipped, but I can say with much confidence the probability is greater than 50%.

Let's assume the mean number of flips is m and the actual number follows the Poisson distribution. In other words, the time between flips follows a memoryless property. I choose this assumption because the actual number of flips can't be less than zero.

The probability of exactly n flips, with a mean of m, is given by the formula $\frac{e^{-m}\, m^n}{n!}$

The following table and chart show the probability of 0 to 25 flips, given a mean of 10.

TOTAL FLIPS	PROBABILITY
0	0.000045
1	0.000454
2	0.002270
3	0.007567
4	0.018917
5	0.037833
6	0.063055
7	0.090079
8	0.112599
9	0.125110
10	0.125110
11	0.113736
12	0.094780
13	0.072908
14	0.052077
15	0.034718
16	0.021699
17	0.012764
18	0.007091
19	0.003732
20	0.001866
21	0.000889
22	0.000404
23	0.000176
24	0.000073
25	0.000029



Remember, the mean number of flips is m. The probability of an even number of flips is:

$$\frac{e^{-m}\, m^0}{0!} + \frac{e^{-m}\, m^2}{2!} + \frac{e^{-m}\, m^4}{4!} + \frac{e^{-m}\, m^6}{6!} + \frac{e^{-m}\, m^8}{8!} + \ldots$$

We can factor out e^{-m} from each term …

$$e^{-m} \times [\frac{m^0}{0!} + \frac{m^2}{2!} + \frac{m^4}{4!} + \frac{m^6}{6!} + \frac{m^8}{8!} + \ldots]$$

Note how this series alternates terms, with only even terms. We need to find a way to introduce odd terms that evaluate to zero.

The humdinger is that 1^n = 1 if n is an even number and 1^n = -1 if n is an odd number.

Note further that $\frac{(1+(-1)^n)}{2}$ = 1 if n is even and 0 if n is odd.

With this trick, we can add odd terms to the series that evaluate to zero.

$$\frac{e^{-m}\, m^0}{0!} \times \frac{(1+(-1)^0)}{2} + \frac{e^{-m}\, m^1}{1!} \times \frac{(1+(-1)^1)}{2} +$$

$$\frac{e^{-m}\, m^2}{2!} \times \frac{(1+(-1)^2)}{2} + \frac{e^{-m}\, m^3}{3!} \times \frac{(1+(-1)^3)}{2} +$$

$$\frac{e^{-m}\, m^4}{4!} \times \frac{(1+(-1)^4)}{2} + \frac{e^{-m}\, m^5}{5!} \times \frac{(1+(-1)^5)}{2} + \ldots$$

We can now express this as an infinite sum:

$$e^{-m} \sum_{n=0}^{\infty} [\ \frac{m^n}{n!} \times \frac{(1+(-1)^n)}{2}\ ]$$

Let's break that into two expressions:

$$e^{-m} \sum_{n=0}^{\infty} [\ \frac{m^n}{n!} \times \frac{1}{2}\ ] + e^{-m} \sum_{n=0}^{\infty} [\ \frac{m^n}{n!} \times \frac{(-1)^n)}{2}]$$

$$= \frac{e^{-m}}{2} \sum_{n=0}^{\infty} \frac{m^n}{n!} + \frac{e^{-m}}{2} \sum_{n=0}^{\infty} \frac{(-m)^n}{n!}$$

Next, let’s review Euler’s equation:

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

So, $\sum_{n=0}^{\infty} \frac{m^n}{n!} = e^{m}$

…and…

$$\sum_{n=0}^{\infty} \frac{(-m)^n}{n!} = e^{-m}$$

Getting back to the original equation for the probability of an even number of flips…

$$= \frac{e^{-m}}{2} \times e^{m} + \frac{e^{-m}}{2} \times e^{-m}$$

$$= \frac{1}{2} + \frac{e^{-2m}}{2}$$

The probability of an odd number of flips is thus…

$$1 - [\frac{1}{2} + \frac{e^{-2m}}{2}] = \frac{1}{2} - \frac{e^{-2m}}{2}$$

Thus, regardless of the mean, the probability a Poisson variable will be even is more than 50% and less than 50% for odd.

The following table shows the probability of an even and odd result for various means.

MEAN	EVEN	ODD
0.5	0.567667642	0.432332358
1	0.509157819	0.490842181
1.5	0.501239376	0.498760624
2	0.500167731	0.499832269
2.5	0.500022700	0.499977300
3	0.500003072	0.499996928
3.5	0.500000416	0.499999584
4	0.500000056	0.499999944
4.5	0.500000008	0.499999992
5	0.500000001	0.499999999